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VOID GROWTH AND COMPACTION RELATIONS FOR DUCTILE POROUS MATERIALS UNDER INTENSE DYNAMIC GENERAL LOADING CONDITIONS

WANG ZE-PING
Institute of Applied Mechanics, Southwest Jiaotong University, Chengdu, Sichuan 610031, People's Republic of China

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Abstract—Void growth and compaction relations for ductile porous materials under intense dynamic general loading conditions are developed. The mathematical models include the influence of inertial effects, material rate sensitivity, as well as the contribution of surface energy of a void and material work-hardening. Numerical analysis shows that inertia appears to resist the growth of voids. The inertial effects increase quickly as the strain rates become higher. The theoretical analysis suggests that the inertial effects cannot be neglected at high strain rates. It is also indicated that the dynamic growth of voids is highly sensitive to the strain rates. The threshold applied external stresses for the void growth and compaction, which are the functions of the distention, are derived from void growth and compaction relations.

1. INTRODUCTION

Dynamic ductile fracture with different loading conditions such as high speed impact, explosive loading, dynamic tension of smooth or notched bar specimens, is a consequence of the nucleation, growth, and coalescence of voids in a triaxial stress field. In comparison to ductile damage under static loading, dynamic ductile damage is much more complex. The inertial effects, rate-dependence and thermal influence from rapid plastic deformation are the characteristics under intense dynamic loading. The literature on this subject is extensive. After investigating the behavior of dynamic damage and fracture in solids in detail, Curran and co-workers (Curran *et al.*, 1977, 1987; Barber *et al.*, 1972; Seaman *et al.*, 1976) established computational models called NAG (nucleation and growth) models for ductile and brittle fracture. In their models, two internal state variables N (the number of microvoids or microcracks per unit volume) and R (the average size of a microvoid or microcrack) are introduced to describe the processes of dynamic damage and fracture in solids. The NAG models have sufficient generality to include the statistical distribution of one or more variables such as porosity, void density, etc., but require numerous phenomenological constants that are difficult to obtain. Carroll and Holt (1972) developed static and dynamic pore-collapse relations for ductile porous materials. The material was assumed to be rate insensitive and ideally plastic. They suggested that the effect of elastic compressibility in the matrix material is small, and can go immediately to the case of fully plastic deformation around the void. Johnson (1981) applied Carroll and Holt's approach to void growth in a viscoplastic medium. Cochran and Banner (1977) studied spallation in uranium using a simple theoretical model. Rajendran *et al.* (1989) proposed a new dynamic failure model to describe void nucleation, growth and coalescence in ductile metals. Nash and Cullis (1984) and Nash (1985) used Rice's static model directly to model ductile fracture in triaxial states of stress. Cortes (1992) adopted Carroll and Holt's assumption to investigate the growth of a microvoid under intense dynamic loading. Review articles (Meyers and Aimone, 1983; Curran *et al.*, 1987; Grady, 1988) on dynamic ductile fracture explain in some detail the most relevant results of experimental and theoretical studies.

The inertial effects are the major feature of intense dynamic loading. Our theoretical analysis shows that its influence on the growth and compaction of dynamic ductile voids is significant. Since the problem is quite complicated, most investigators have neglected it.

The influence of deviatoric stresses is also important for most of the processes of dynamic ductile fracture. Although in the past several authors have studied the void growth problem under triaxiality conditions (Rice and Tracey, 1969; Gurson, 1977; Duva and Hutchinson, 1984; Cocks, 1989) they have limited their analysis to static loading, ignoring the influence of inertial effects. In this paper, we deal with dynamic growth and compaction of voids in ductile materials under extremely high rates of general loading. Void growth and compaction relations, in which the inertial effects, rate-dependence and the contribution of surface energy of a void are considered, are presented by means of the energy principles.

To simplify theoretical analysis, we assume that the matrix material is incompressible during the void growth and compaction. We also assume that the void remains spherical all the time. These assumptions lead to a great simplification of the theoretical analysis, so that we can obtain the exact relations for both void growth and compaction.

2. VOID GROWTH AND COMPACTION RELATIONS

The problem analysed in this paper is shown diagrammatically in Fig. 1. We assume that the porous material consists of a suspension of pores in a matrix of homogeneous isotropic solid ductile material which is subjected to an external stress Σ_{ij}, and that the porous material is statistically homogeneous and isotropic so that it can be effectively modeled by a homogeneous isotropic solid material. With these assumptions, we can study the void growth and compaction by considering a hole sphere of the matrix material of inner radius a and outer radius b [Fig. 1(b)]. Distention α is defined as

$$\alpha = \frac{b^3}{b^3 - a^3}. \tag{1}$$

We investigate the response of this hole sphere to time-dependent external stress and zero internal pressure, and try to obtain the relations between the applied stress Σ_{ij} and distention $\alpha(t)$. We expect that these relations will adequately describe the void growth and compaction for the effective homogeneous material.

Taking the matrix material and void as a system, work done by the applied external stress Σ_{ij} is equal to the change of the system energy, namely,

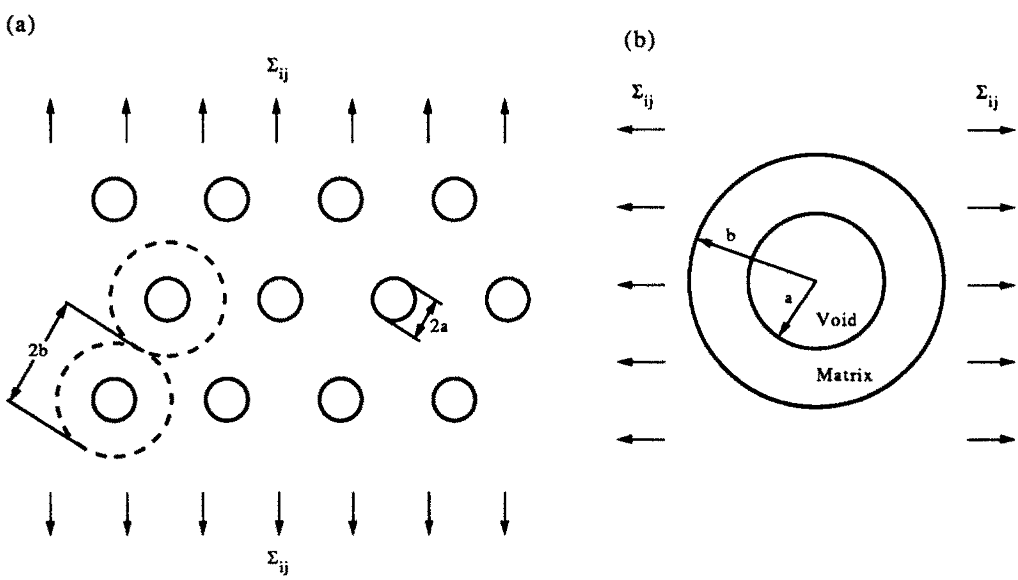


Fig. 1. (a) A body containing a homogeneous distribution of spherical voids of mean radius a and mean spacing b is subjected to an applied external stress Σ_{ij}. (b) A spherical element of material of radius b containing a single void of radius a.

$$\Delta E_K + \Delta E_S + \Delta E_i = W \tag{2}$$

where ΔE_K, ΔE_S, ΔE_i denote the changes of the kinetic energy E_K, the surface energy of a void E_S, and the internal energy E_i in the system respectively. W denotes the work done by the applied external stress Σ_{ij}. As mentioned in the previous section, we neglect the initial elastic and elastic–plastic phases of the process, and begin to consider fully plastic deformation in the solid around the void. From the assumption of incompressibility in the matrix material surrounding the void and eqn (2), the following expressions can be derived (Johnson, 1981)

$$r^3 = r_0^3 - B(t) \tag{3}$$

$$B(t) = a_0^3 \frac{\alpha_0 - \alpha}{\alpha_0 - 1} \tag{4}$$

$$\frac{B(t)}{a^3} = \frac{\alpha_0 - \alpha}{\alpha - 1} \tag{5}$$

$$\frac{B(t)}{b^3} = \frac{\alpha_0 - \alpha}{\alpha} \tag{6}$$

where r is the Eulerian radial position of a Lagrangian point that travels with the material. The initial radius of that point is r_0. $B(t)$ is a function related to the rate of void growth. a_0 is the initial radius of the void.

Consider ΔE_K, ΔE_S, ΔE_i and W, respectively. ΔE_K is given by

$$\Delta E_K = E_K(\alpha) - E_K(\alpha_0) \tag{7}$$

with

$$E_K(\alpha) = \int_a^b \tfrac{1}{2}\dot{r}^2 \rho 4\pi r^2 \, dr \tag{8}$$

where "$\cdot$" denotes the differential with respect to time t. ρ is the density of the matrix material. With the use of eqn (1) and eqns (3)–(6), eqn (8) becomes

$$E_K(\alpha) = \left[\frac{4\pi a_0^3}{9(\alpha_0 - 1)}\right] \frac{a_0^2 \rho}{2(\alpha_0 - 1)} \left(\frac{\alpha_0 - 1}{\alpha - 1}\right)^{1/3} \left[1 - \left(\frac{\alpha - 1}{\alpha}\right)^{1/3}\right] \dot{\alpha}^2. \tag{9}$$

In the same way, we can also obtain the expression of ΔE_S,

$$\Delta E_S = \pm [E_S(\alpha) - E_S(\alpha_0)] \tag{10}$$

with

$$E_S(\alpha) = \left[\frac{4\pi a_0^3}{9(\alpha_0 - 1)}\right] \frac{9(\alpha_0 - 1)^{1/3}\gamma}{a_0} (\alpha - 1)^{2/3} \tag{11}$$

where γ is the surface energy expended per unit area during the hole expanding. The upper sign (+) corresponds to void growth and the lower sign (−) corresponds to void compaction. Note that in the following sections all the signs $\pm$ correspond to void growth and compaction, respectively, unless they are explained again.

The problem studied in the present paper is mainly concerned with the dynamic growth and compaction of a spherical void under intense dynamic loading. The dynamic growth and compaction of a ductile void can be approximately considered as an adiabatic process. In addition, we suppose that there is no heat source in the system. With these assumptions, the increment of the specific internal energy is given by

$$de_i = \frac{1}{\rho}\sigma_{jk}d\varepsilon_{jk} = \frac{1}{\rho}(\sigma_{jk}d\varepsilon^{e}_{jk} + \sigma_{jk}d\varepsilon^{p}_{jk}) \tag{12}$$

where σ_{jk} is the stress tensor component, and ε_{jk} is the strain tensor component. Superscripts "e" and "p" denote elastic and plastic parts of strain tensor component. The total strain increment is assumed to be the sum of elastic and plastic contributions:

$$d\varepsilon_{jk} = d\varepsilon^{e}_{jk} + d\varepsilon^{p}_{jk}. \tag{13}$$

Based on the results that effects of elastic deformation in the solid around the void is much smaller than that of plastic deformation, the term $(1/\rho)\ \sigma_{jk}d\varepsilon^{e}_{jk}$ in eqn (12) can be ignored. Then eqn (12) can be written as

$$\begin{aligned} de_i &= \frac{1}{\rho}\sigma_{jk}d\varepsilon^{p}_{jk} \\ &= \frac{1}{\rho}\left(\sigma'_{jk} + \frac{1}{3}\sigma_{mm}\delta_{jk}\right)d\varepsilon^{p}_{jk} \\ &= \frac{1}{\rho}\left(\sigma'_{jk}\, d\varepsilon^{p}_{jk} + \frac{1}{3}\sigma_{mm}\delta_{jk}d\varepsilon^{p}_{jk}\right) \end{aligned} \tag{14}$$

where σ'_{jk} is the deviatoric stress tensor component. Since we assume that the matrix material is incompressible, namely, $d\varepsilon^{p}_{kk} = 0$, eqn (14) is, therefore, reduced to

$$de_i = \frac{1}{\rho}\sigma'_{jk}\, d\varepsilon^{p}_{jk} = \frac{1}{\rho}\sigma_{eqv}\, d\varepsilon^{p}_{eqv} \tag{15}$$

where σ_{eqv} and ε^{p}_{eqv} denote the effective stress and the effective plastic strain in the matrix material.

The matrix material is assumed to be linear work-hardening, and viscoplastic. The constitutive relation is supposed to be (Perzyna, 1986)

$$\sigma_{eqv} = Y_0 + H\varepsilon^{p}_{eqv} + \eta\dot{\varepsilon}^{p}_{eqv} \tag{16}$$

where Y_0 is the yield stress of the matrix material, H is a linear work-hardening coefficient, and η is the material viscosity. Since we assume a plastic deformation process with spherical symmetry, the effective plastic strain ε^{p}_{eqv} is given by Johnson and Mellor (1973)

$$\varepsilon^{p}_{eqv} = \pm 2\ln\frac{r}{r_0}. \tag{17}$$

The change of internal energy in the system is

$$\Delta E_i = \frac{1}{\rho}\int_a^b \left[\int_0^{\varepsilon^{p}_{eqv}} \sigma_{eqv}(\varepsilon^{p})d\varepsilon^{p}\right] 4\pi\rho r^2\, dr. \tag{18}$$

By means of eqns (3)–(6) and eqn (16), we finally have

$$\Delta E_i = \left[\frac{4\pi a_0^3}{9(\alpha_0-1)}\right][F_3(\alpha)+F_4(\alpha)+F_5(\alpha)\dot{\alpha}] \tag{19}$$

where

$$F_3(\alpha) = \pm 2Y_0\left(\ln\frac{\alpha-1}{\alpha_0-1}+\alpha\ln\frac{\alpha}{\alpha-1}-\alpha_0\ln\frac{\alpha_0}{\alpha_0-1}\right) \tag{20}$$

$$F_4(\alpha) = \tfrac{4}{3}H\left[(\alpha-\alpha_0)F(\alpha)+\frac{\alpha_0}{2}\left(\ln\frac{\alpha_0}{\alpha}\right)^2-\frac{\alpha_0-1}{2}\left(\ln\frac{\alpha_0-1}{a-1}\right)^2\right] \tag{21}$$

$$F(\alpha) = \int_{h_0}^{h_1}\frac{\ln(h+1)}{h}\,\mathrm{d}h \tag{22}$$

with

$$h_0 = \frac{\alpha_0-\alpha}{\alpha-1},\quad h_1 = \frac{\alpha_0-\alpha}{\alpha}$$

$$F_5(\alpha) = \tfrac{2}{3}\eta\left(\ln\frac{\alpha-1}{\alpha}+\ln\frac{\alpha_0}{\alpha_0-1}\right). \tag{23}$$

Functions $F_3(\alpha)$, $F_4(\alpha)$ and $F_5(\alpha)$ denote the influence of the yield stress of the matrix material, the linear work-hardening and the material viscosity on the increment of internal energy, respectively. The increment of work per unit volume done by the applied external stress Σ_{ij} is given by

$$\mathrm{d}w = \Sigma_{ij}\mathrm{d}E_{ij} \tag{24}$$

where E_{ij} is the macroscopic deformation component which is defined in the same way as Gurson's (1977) definition

$$\dot{E}_{ij} = \frac{1}{V}\int_V \dot{\varepsilon}_{ij}\,\mathrm{d}V \tag{25}$$

where V is the volume of the spherical element, and $\dot{\varepsilon}_{ij}$ is the microscopic rate of the deformation field. With external boundary conditions put in terms of the $\dot{E}_{ij}$,

$$v_i|_S = \dot{E}_{ij}x_j|_S \quad \text{(Cartesian coordinates)} \tag{26}$$

where v_i is the microscopic velocity field, and x_i is the position of a material point in Cartesian coordinates. From eqns (25)–(26), we have

$$\dot{E}'_{ij} = \dot{\varepsilon}'_{ij},\quad \dot{E}_{kk} = \dot{V}/V \tag{27}$$

where $\dot{E}_{ij}$ and $\dot{\varepsilon}'_{ij}$ are the deviatoric parts of the macroscopic and microscopic rates of deformation field, respectively. The increment of work per unit volume also may be written

$$\begin{aligned}\mathrm{d}w &= \Sigma_{ij}\,\mathrm{d}E_{ij}\\ &= (\Sigma'_{ij}+\tfrac{1}{3}\Sigma_{kk}\delta_{ij})(\mathrm{d}E'_{ij}+\tfrac{1}{3}\delta_{ij}\,\mathrm{d}E_{nn})\\ &= \Sigma'_{ij}\,\mathrm{d}E'_{ij}+\tfrac{1}{3}\Sigma_{kk}\,\mathrm{d}E_{nn}\\ &= \Sigma_{\mathrm{eqv}}\,\mathrm{d}E_{\mathrm{eqv}}+\tfrac{1}{3}\Sigma_{kk}\frac{\mathrm{d}V}{V}\end{aligned} \tag{28}$$

where Σ'_{ij} is the macroscopic deviatoric stress tensor component, Σ_{eqv} and E_{eqv} denote the macroscopic effective stress and strain,

$$\Sigma_{eqv} = (\tfrac{3}{2}\Sigma'_{ij}\Sigma'_{ij})^{1/2},\quad E_{eqv} = (\tfrac{2}{3}E'_{ij}E'_{ij})^{1/2},\quad \tfrac{1}{3}\Sigma_{kk} = -P. \tag{29}$$

Using eqns (5)–(6) and (27), the following relations are given

$$\mathrm{d}E_{eqv} = d\varepsilon^{p}_{eqv} = \frac{2}{3}\frac{\mathrm{d}\alpha}{\alpha},\quad \frac{\dot{V}}{V} = \frac{\dot{\alpha}}{\alpha}. \tag{30}$$

With the help of eqns (28)–(30), work W done by external stress is as follows:

$$W = \int_{\alpha_0}^{\alpha} \tfrac{4}{3}\pi b^3\,\mathrm{d}w = \tfrac{4}{3}\pi\frac{a_0^3}{\alpha_0 - 1}\int_{\alpha_0}^{\alpha} [\tfrac{2}{3}\Sigma_{eqv}(\alpha') - P(\alpha')]\,\mathrm{d}\alpha'. \tag{31}$$

Here we assume Σ_{eqv} and P to be functions of distention α, that is, $\Sigma_{eqv} = \Sigma_{eqv}(\alpha)$ and $P = P(\alpha)$. Substitution of eqns (7)–(11), eqn (19) and eqn (31) into eqn (2) gives

$$F_1(\alpha)\dot{\alpha}^2 + F_5(\alpha)\dot{\alpha} + F_7(\alpha) = 0 \tag{32}$$

where

$$F_1(a) = \frac{\rho a_0^2}{2(\alpha_0 - 1)}\left(\frac{\alpha_0 - 1}{\alpha - 1}\right)^{1/3}\left[1 - \left(\frac{\alpha - 1}{\alpha}\right)^{1/3}\right] \tag{33}$$

$$F_7(\alpha) = F_2(\alpha) + F_3(\alpha) + F_4(\alpha) - 3\int_{\alpha_0}^{\alpha} [\tfrac{2}{3}\Sigma_{eqv}(\alpha') - P(\alpha')]\,\mathrm{d}\alpha' - F_1(\alpha_0)\dot{\alpha}_0^2 - F_2(\alpha_0) \tag{34}$$

$$F_2(\alpha) = \pm\frac{9(\alpha_0 - 1)^{1/3}\gamma}{a_0}(\alpha - 1)^{2/3}. \tag{35}$$

Equation (32) is the relationship from which we obtain the rate-dependent response of the void growth and compaction under dynamic loading. $F_2(\alpha)$ represents the influence of change of the surface energy of a void on the void growth and compaction. The terms in eqn (32) have a clear physical significance. The first term $F_1(\alpha)\dot{\alpha}^2 - F_1(\alpha_0)\dot{\alpha}_0^2$ on the left of eqn (32) represents inertial resistance to the void growth and compaction. The second term $F_5(\alpha)\dot{\alpha}$ denotes the influence of the material viscosity, which describes the effect of the rate sensitivity, and is one of the major features differing from the quasi-static growth and compaction of voids. Other researchers' studies, such as those by Curran *et al.* (1987), Johnson (1981) and Cortes (1992), have given the same result. The third term $F_7(\alpha)$ is the total effect of the applied external stress, the change of the surface energy of voids, work-hardening and the yield stress in the solid surrounding the void on the void growth and compaction. These effects also can be isolated and studied in great depth. Dynamic ductile fracture is a consequence of the nucleation, growth and coalescence of voids in a triaxial stress. Besides the mean stress, the deviatoric stress, no doubt, may affect the void growth and compaction. The influence of the deviatoric stress (or the deviatoric strain) on the void growth is considered in Rice and Tracey's model (1969) and Gurson's model (1977). A modified Gurson's model was successfully applied to model the cup–cone fracture in a round tensile bar (Tvergaard and Needleman, 1984). Unfortunately, the effect of the deviatoric stress was not included in Carroll and Holt's (1972) nor the Seaman *et al.* (1976) models of dynamic ductile fracture. From eqn (32), the void growth and compaction rates $\dot{\alpha}$ can be given

$$\dot{\alpha} = \frac{1}{2F_1(\alpha)}\{-F_5(\alpha) \pm \sqrt{[F_5(\alpha)]^2 - 4F_1(\alpha)F_7(\alpha)}\} \tag{36}$$

where $F_1(\alpha) > 0$, $F_5(\alpha) > 0$ (void growth), $F_5(\alpha) < 0$ (void compaction).

If the inertial effects are neglected, from eqn (32), $\dot{\alpha}$ is reduced to

$$\dot{\alpha} = -[F_7(\alpha) + F_1(\alpha_0)\dot{\alpha}_0^2]/F_5(\alpha). \tag{37}$$

Equation (37) is suitable for both void growth and compaction. However, one must note that $F_5(\alpha) < 0$ for the void compaction.

3. THRESHOLD STRESSES FOR DYNAMIC GROWTH AND COMPACTION OF VOIDS

Define a quantity Σ as

$$\Sigma = \tfrac{2}{3}\Sigma_{eqv} - P. \tag{38}$$

Obviously, Σ represents the total external stress acting on the spherical element. It shows that either the mean stress $-P$ or the effective stress Σ_{eqv} has a contribution to the void growth and compaction. We first consider the condition of the void growth, namely, $\dot{\alpha} \geqslant 0$. Analysis of the condition of the void compaction is the same as that of the void growth. From eqn (36), the following inequality must be satisfied:

$$\Sigma(\alpha) \geqslant \frac{1}{3}\left[\frac{dF_2(\alpha)}{d\alpha} + \frac{dF_3(\alpha)}{d\alpha} + \frac{dF_4(\alpha)}{d\alpha}\right]. \tag{39}$$

With the help of eqns (20)–(21) and eqn (35), inequality (39) becomes

$$\Sigma(\alpha) \geqslant \frac{1}{3}\left[\frac{6\gamma}{a_0}\left(\frac{\alpha_0-1}{\alpha-1}\right)^{1/3} + \frac{4H}{3}F(\alpha) - \frac{8H}{3}\frac{\alpha_0-1}{\alpha-1}\ln\frac{\alpha-1}{\alpha_0-1} + 2Y_0\ln\frac{\alpha}{\alpha-1}\right]. \tag{40}$$

By analogy with the condition of void growth, we can also obtain an inequality for the void compaction,

$$\Sigma(\alpha) \leqslant -\frac{1}{3}\left[\frac{6\gamma}{a_0}\left(\frac{\alpha_0-1}{\alpha-1}\right)^{1/3} + \frac{4H}{3}F(\alpha) - \frac{8H}{3}\frac{\alpha_0-1}{\alpha-1}\ln\frac{\alpha-1}{\alpha_0-1} + 2Y_0\ln\frac{\alpha}{\alpha-1}\right]. \tag{41}$$

Let

$$\Sigma_{crit}(\alpha) = \pm\frac{1}{3}\left[\frac{6\gamma}{a_0}\left(\frac{\alpha_0-1}{\alpha-1}\right)^{1/3} + \frac{4H}{3}F(\alpha) - \frac{8H}{3}\frac{\alpha_0-1}{\alpha-1}\ln\frac{\alpha-1}{\alpha_0-1} + 2Y_0\ln\frac{\alpha}{\alpha-1}\right], \tag{42}$$

where $\Sigma_{crit}(\alpha)$ are the threshold stresses for both dynamic growth and compaction of voids in general dynamic loading conditions. Sign $(+)$ is for void growth and sign $(-)$ is for void compaction. The critical conditions that the applied external stress must satisfy for both void growth and compaction are that

$$\begin{cases} \Sigma(\alpha) > \Sigma_{\rm crit}(\alpha) & \text{(void growth)} \\ \Sigma(\alpha) < \Sigma_{\rm crit}(\alpha) & \text{(void compaction).} \end{cases} \tag{43}$$

If the contributions of the change of surface energy and material work-hardening, as well as the action of the external deviatoric stress are neglected, eqn (42) is reduced to

$$P_{\rm crit}(\alpha) = \pm \tfrac{2}{3} Y_0 \ln \frac{\alpha}{\alpha - 1} \tag{44}$$

where the upper sign (+) corresponds to void compaction and the lower sign (−) corresponds to void growth. This is the result obtained by Carroll and Holt (1972).

4. NUMERICAL ANALYSIS

In this section, pure copper is selected to be the material for the numerical analysis with density $\rho = 8.92$ g/cm^3, yield stress $Y_0 = 0.26$ GPa, linear work-hardening coefficient $H = 0.25$ GPa, viscosity $\eta = 0.1$ GPa · μs, and surface energy expended per unit area $\gamma = 0.9$ kJ/m^2. In addition, only the situation of void growth is considered. The approach to analysis of the void compaction is analogous.

The relation of the threshold stress $\Sigma_{\rm crit}(\alpha)$ and distention α for the void growth in terms of eqn (42) is depicted in Fig. 2. It shows that the threshold stress $\Sigma_{\rm crit}(\alpha)$ decreases quickly as distention α increases. The maximum value of the threshold stress $\Sigma_{\rm crit}(\alpha)$ is about 1.76 GPa which depends on the initial radius of the void a_0 and the initial distention α_0. Our experimental observation (Wang, 1993) shows that the initial radius of the void a_0 is in the range of 1–10 μm, and a_0 and α_0 are 0.0005 cm and 1.0003. The spall experimental results in copper (Grady, 1988) show that the spall strength of copper is in the range 1.0–2.5 GPa. Theoretical calculation in this paper is consistent with the experimental measurements. The threshold stress of the dynamic fracture of the void is larger than that of the quasi-static fracture. This may be due to the inertia and the kinetics associated with the micromechanisms controlling the damage process. This implies that the larger the ductile voids in the solid, the smaller the stress needed for void growth.

To investigate the effects of inertia and viscosity (or rate-dependency) on the behavior of the void growth and compaction under different rate loading, we numerically analyse the analytical eqns (36)–(37) previously developed. In order to simplify the analysis, the material is assumed to be subjected to a linearly increasing external stress,

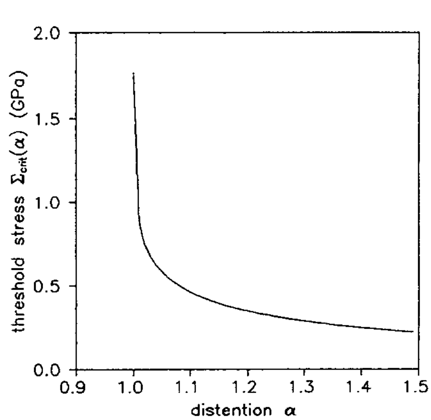


Fig. 2. The threshold stress $\Sigma_{\rm crit}(\alpha)$ for the void growth decreases along with increase of distention α.

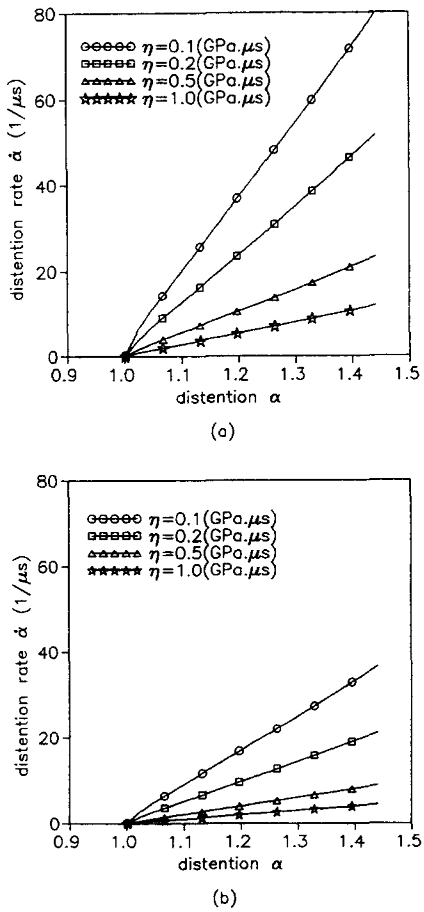


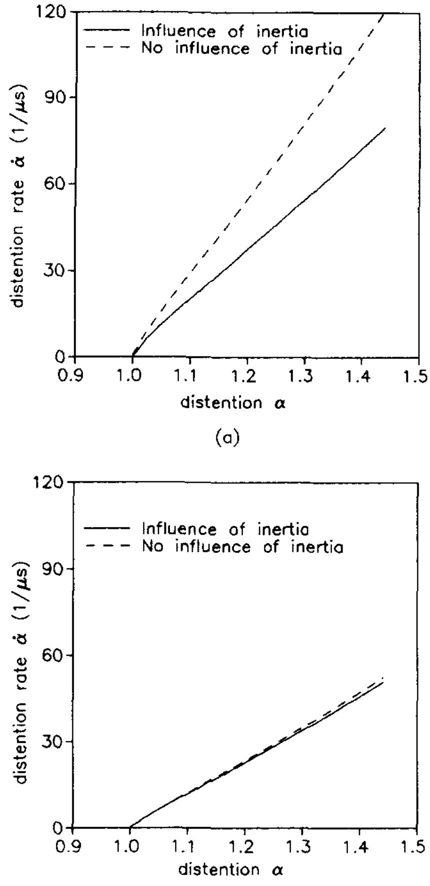
Fig. 3. Influence of material viscosity η with different values on the rate of distention α under the different loading rates. (a) $G = 0.3$, (b) $G = 0.1$.

$$\Sigma(\alpha) = \Sigma_0 + G(\alpha - \alpha_0) \tag{45}$$

where $\Sigma_0 = \Sigma_{crit}(\alpha_0)$ and G is a constant.

Theoretical analysis and experimental observations (Johnson, 1981; Curran *et al.*, 1987; Cortes, 1992; Wang, 1993) have evidenced that the material viscosity (rate-dependent sensitivity), especially in the high strain rate range, has a great influence on the process of damage and fracture in solids. The mathematical model of dynamic growth of a void presented in the present work also shows the same result. In general, the material viscosity η is considered proportional to $1/\sqrt{\dot{\varepsilon}}$ (Chhabildas and Asay, 1979; Cortes, 1992), where $\dot{\varepsilon}$ denotes strain rate. This means that the higher the strain rate in the material, the lower the material viscosity. We may estimate that for $\dot{\varepsilon} \sim 10^3\ s^{-1}$, $\eta \sim 1.0$ GPa · μs, and for $\dot{\varepsilon} \sim 10^5\ s^{-1}$, $\eta \sim 0.1$ GPa · μs. In Fig. 3, two sets of curves are given in terms of eqn (36). Profiles show the changes of distention rate $\dot{\alpha}$ along with α for different viscosity value η under the different rates of the applied external stress Σ. It is obvious that the effects of different viscosity η on the change of $\dot{\alpha}$ along with distention α is great. $\dot{\alpha}$ also rises quickly as the loading rates increase. These important results indicate that the material viscosity plays a very important role in the behavior of dynamic ductile damage of the solids, especially for high loading rates ($\dot{\varepsilon} \geqslant 10^3\ s^{-1}$).

Figure 4 indicates that the influence of inertial effects becomes larger along with the increase of loading rates. The results of numerical analysis in Fig. 4 suggest that the influence of inertial effects plays an important role in the void growth under high rate loading conditions. On the other hand, the numerical analysis in Fig. 4(b) also shows the fact that if the rate of applied external stress becomes much lower, the inertial effects can be ignored.



(b)

Fig. 4. Influence of inertia on the rate of distention α under the different loading rates. (a) $G = 0.3$, (b) $G = 0.01$.

This result implies that the inertial effects are a mechanical phenomenon which appear significant in the condition of intense dynamic loading.

5. DISCUSSION

Many experimental observations (Meyers and Aimone, 1983; Curran *et al.*, 1987; Wang, 1993) show that the dynamic ductile fracture in solids is a complicated process which, in general, involves nucleation, growth and coalescence of microvoids. The processes of damage in the materials are not the same due to different loading conditions as well as different microstructures of the materials. Each stage of the nucleation, growth and coalescence of microvoids is also complex. In comparison with the quasi-state condition, the dynamic growth of a void presents some additional complications:

(a) The heat generated by plastic deformation cannot dissipate itself due to the high rate of deformation.

(b) Inertial effects associated with the displacement of the material adjoining the void walls become an important consideration.

(c) Rate sensitivity of the materials (or the material viscosity) is caused by intense dynamic loading.

The behavior of the dynamic ductile fracture is so complex that in order to obtain a mathematical model, some assumptions must be utilized. In this paper, we assume that the matrix material surrounding voids is incompressible and the void is spherical during pore growth. These assumptions afford a great simplification of the theoretical analysis.

The inertial effects are indeed important for the dynamic fracture in the material under high strain rates. Although they have not studied the problem in depth, many investigators have been paying attention to the influence of inertial effects on the dynamic ductile fracture (Carroll and Holt, 1972; Johnson, 1981; Meyers and Aimone, 1983; Tvergaard and Needleman, 1986; Cortes, 1992). Experimental observations of crack propagation in ductile metals seem to show that the maximum velocity is much lower than in the case of brittle fracture (Glennie, 1972). Glennie attributes this reduction in the maximum velocity to inertial effects and provides an approximate solution for the growth of voids taking into account these inertial effects. The numerical analysis in this paper shows that inertial effects appear to resist the void growth. It increases quickly as the loading rates are enhanced.

Material viscosity (or rate sensitivity), especially in the high strain rate range, has a great influence on the void growth, which is shown by the numerical analysis in the previous section. The work of Curran *et al.* (1987), Johnson (1981) and Cortes (1992) showed the same result. Effect of material rate-dependence is one of the major factors which influence the behavior of dynamic ductile fracture. Our theoretical analysis shows that the higher the strain rate, the greater the effect of rate-dependence.

During the process of most dynamic ductile fractures, such as spallation, the void growth is mainly controlled by tensile stress. But it is found that deviatoric stress (or shear strains) are important for some dynamic ductile fractures, for example in the cylinder test (Nash, 1985), and the dynamic tension test (Johnson, 1988). The fracture of adiabatic shear bands which, in some cases, is composed of small voids, is attributed to shear deformation or equally deviatoric stresses. Some investigators such as Rice and Tracey (1969), Gurson (1977) and Cocks (1989) have considered the action of deviatoric stresses on the void growth. Unfortunately, they only considered the static loading. In effect, the action of the deviatoric stress still exists. Up to now most models of dynamic ductile fracture have not taken the effect of the deviatoric stress into account (Carroll and Holt, 1972; Seaman *et al.*, 1976; Johnson, 1981; Cortes, 1992). Both the deviatoric and mean stresses are, in our work, taken into account in the models for the dynamic growth and compaction of voids in the ductile porous materials.

6. CONCLUSIONS

Void growth and compaction relations in the ductile porous materials under dynamic general loading conditions are presented by means of the energy principles with the assumption that the matrix materials are incompressible and the void is spherical during the void growth or compaction. The influence of inertial effects and material rate sensitivity, as well as the contribution of surface energy of a void and material work-hardening are taken into account in the relations. One of the major differences in the model proposed in this work, as compared to other microscopic descriptions of dynamic ductile damage (Seaman *et al.*, 1976; Carroll and Holt, 1972; Johnson, 1981; Cortes, 1992), is that not only the external tensile pressure P, but also the external deviatoric stress Σ_{eqv} (the effective stress) are taken into account for the void growth and compaction. The threshold stresses (the combined action of hydrostatic and deviatoric stresses) as functions of the current distention are directly obtained. Another feature of the model is that the influence of the inertial effects on the void growth is included. Numerical analysis suggests that the inertial effects appear to resist the growth of voids. It is significant at high strain rates and cannot be neglected. The effect of rate sensitivity of the materials, which intensively influences the behavior of the void growth and compaction, is also involved in the relations.

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